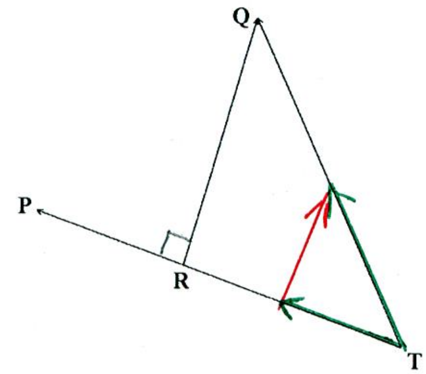
Let P be the point $(1, -4, 7)$, Q be the point $(-6, -1, 2)$, and T be the point $(-1, 5, -3)$ in the diagram on the right.

SCORE: _____ / 95 PTS

NOTE: The diagram is NOT drawn to scale.



[a] In the diagram, sketch the vector $\frac{1}{2}\overrightarrow{TQ} - \frac{1}{3}\overrightarrow{TP}$.

[b] Find $\overrightarrow{RQ}$.

$$\overrightarrow{TR} = \mathrm{PROJ}_{\overrightarrow{TP}} \overrightarrow{TQ} = \mathrm{PROJ}_{\langle 2,-9,10\rangle} \langle -5,-6,5\rangle$$

$$= \frac{\langle -5,-6,5\rangle \cdot \langle 2,-9,10\rangle}{\langle 2,-9,10\rangle \cdot \langle 2,-9,10\rangle} \langle 2,-9,10\rangle$$

$$= \frac{94}{185} \langle 2,-9,10\rangle = \left\langle \frac{188}{185}, \frac{-846}{185}, \frac{940}{185} \right\rangle$$

$$\overrightarrow{RQ} = \overrightarrow{TQ} - \overrightarrow{TR} = \langle -5,-6,5\rangle - \left\langle \frac{188}{185}, \frac{-846}{185}, \frac{940}{185} \right\rangle = \left\langle \frac{-1113}{185}, \frac{-264}{185}, \frac{-3}{37} \right\rangle$$

[c] Find the general equation of the plane passing through P, Q and T.

$$\overrightarrow{TQ} \times \overrightarrow{TP} = \langle -5,-6,5\rangle \times \langle 2,-9,10\rangle$$

$$= \langle -15, 60, 57\rangle \quad \text{USE } \vec{n} = \langle 5, -20, -19\rangle$$

$$5(x-1) - 20(y+4) - 19(z-7) = 0$$

$$5x - 20y - 19z + 48 = 0$$

[d] Find the volume of the parallelepiped with $\overrightarrow{TQ}$, $\overrightarrow{TP}$ and $\overrightarrow{TS}$ as adjacent sides, if $\overrightarrow{TS} = 3\vec{j} - 2\vec{k}$ (not shown in the diagram).

$$\overrightarrow{TS} \cdot (\overrightarrow{TQ} \times \overrightarrow{TP}) = \langle 0,3,-2\rangle \cdot \langle -15,60,57\rangle = 66$$

CONTINUED FROM PREVIOUS PAGE

[e] Find the co-ordinates of S (as described in [c]).

$\langle 0,3,-2\rangle = \langle x+1, y-5, z+3\rangle$

$x=-1$

$y=8$ $\quad\quad (-1,8,5)$

$z=-5$

[f] Find a vector of magnitude 4 in the opposite direction as $\overrightarrow{TQ}$.

$-4\left(\frac{1}{\|\langle -5,-6,5\rangle\|}\right)\langle -5,-6,5\rangle = \frac{-4}{\sqrt{86}}\langle -5,-6,5\rangle$

$= \left\langle \frac{20}{\sqrt{86}}, \frac{24}{\sqrt{86}}, \frac{-20}{\sqrt{86}} \right\rangle$

[g] Find symmetric equations for the line passing through T and parallel to the line $\begin{matrix} x = 8t-6 \\ y = t+9 \\ z = 4-2t \end{matrix}$.

$\vec{d} = \langle 8,1,-2\rangle$

$\frac{x+1}{8} = y-5 = \frac{z+3}{-2}$

[h] Find parametric equations for the line passing through Q and perpendicular to both the plane $2x-3y-z-9=0$ as well as the plane $-4x+6y+2z+7=0$.

$\vec{n_1} = \langle 2,-3,-1\rangle \quad \vec{n_2} = \langle -4,6,2\rangle = -2\vec{n_1}$

PLANES ARE PARALLEL

$\vec{d} = \vec{n_1}$

$x = -6+2t$

$y = -1-3t$

$z = 2-t$

Which octant or octants contain all points (x, y, z) where $x < 0$ and $yz < 0$? **SCORE: _____ / 15 PTS**
(Both conditions have to be true simultaneously.)

$x<0, y<0, z>0 \longrightarrow O_3$

$x<0, y>0, z<0 \longrightarrow O_{2+4} = O_6$

Two forces are applied to an object. **SCORE: _____ / 40 PTS**

The first force is represented by the vector $\vec{F_1} = < -3, 6 >$.

The second force is represented by the vector $\vec{F_2}$ with direction angle $\frac{7\pi}{6}$ such that $\|\vec{F_2}\| = 10$.

[a] Find the direction angle of $\vec{F_1}$. (Your answer should be in radians, rounded to 2 decimal places.)

$\|\vec{F_1}\| = 3\sqrt{5}$

$\Theta = \cos^{-1} \frac{-3}{3\sqrt{5}} = 2.03$

$\text{OR } \pi - \sin^{-1} \frac{6}{3\sqrt{5}}$

$\text{OR } \pi + \tan^{-1} \frac{6}{-3}$

[b] Find the resultant of the two forces. Write your final answer as a linear combination of $\vec{i}$ and $\vec{j}$.
Do NOT use decimal approximations.

$\langle -3, 6 \rangle + \langle 10 \cos \frac{7\pi}{6}, 10 \sin \frac{7\pi}{6} \rangle$

$= \langle -3, 6 \rangle + \langle -5\sqrt{3}, -5 \rangle$

$= (-3 - 5\sqrt{3})\vec{i} + \vec{j}$

[c] If the resultant of the two forces moves the object from $(3, -8)$ to $(-1, -2)$, find the work done.
Do NOT use decimal approximations.

$\langle -3 - 5\sqrt{3}, 1 \rangle \cdot \langle -4, 6 \rangle$

$= 12 + 20\sqrt{3} + 6$

$= 18 + 20\sqrt{3}$